# A Strict Examination of Cantor's Infinities

## Preamble

This paper must begin with an apology. Out of some mixture of guilt, vanity, and hubris, I believed I could present this paper in a fun way, emphasizing the wonders of new knowledge and musing on the eternal march of progress. After a lot of reflection, I reworked it to give it the sense of decorum that it deserves.

The fact of the matter is that, if I'm right, Cantor's rich and vibrant infinities might be replaced with a system with limited potential for further exploration. A hundred years of beautiful mathematics may be wiped out. A fun presentation was meant to soften the blow, but I don't think it's going to be fun for any mathematicians who have to see their work cast aside, to say nothing of Cantor's descendants.

As far as why I would publish this paper in the first place, I wish I could say it came from some higher sense of duty in contributing to the body of human knowledge. The final tipping point was the fact that I need to take care of my mother, who is disabled. I'm also disabled, but in a number of ways that are hard to prove. The only jobs I can get are the kind that exacerbate my symptoms and our savings are running out fast. I'm hoping that even if I'm wrong, or if this ground has been tread before, someone

can appreciate my imagination enough to award me with some kind of scholarship, career change, or at least help with my debt.

This may seem like a valid reason to some, but the reality is that I'm still trying to commoditize something that may destroy the work of a lot of mathematicians, past and present, and I'm doing it for selfish reasons.

I am truly sorry for any harm this paper will cause.

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## Abstract

Georg Cantor's methodology and proofs will be shown to be ineffective at gauging the sizes of infinities via counterexample. The closure property of the natural numbers will be falsified. The natural numbers will be shown to be more accurately understood as a class. Internally consistent methods of measuring and navigating infinite sets will be demonstrated. The consequences of this paper's findings will then be discussed.

## Introduction

No one will convince me that Georg Cantor was anything less than a titan. He started a revolution in mathematics, and not just in set theory. Everything from number theory to topology carries his influence.

His model of infinite sets, however, has many paradoxes and counterintuitive behaviors. This paper will argue that the paradoxes and behaviors in this system are not the results of the naturally paradoxical behavior of infinite sets, but instead are an indication that our current understanding of the behaviors of infinite sets is fundamentally wrong. This paper will also show that doing away with the closure property of the naturals will resolve every issue brought up here.

I know that I'm making some very bombastic claims. I only ask that you read the paper from the beginning to end so that you can at least understand my reasoning, even if I turn out to be wrong. Skipping to the end will only cause you to wonder why I'm using constructivist methods on mathematical objects that are usually explored formalistically. I do recognize that this is a fifty page proof, however, so if you must skip somewhere, please go to the section titled "Counting the Real Numbers" first.

As one last note, I was unable to write this paper in a standard academic format. My thoughts and mistakes as a layperson inform the path I take in this exploration so I need to tell this as a story. I can only hope that neither this confession nor the narrative form of this paper cause me to be preemptively dismissed as being unserious.

## Exploration

While in college, I heard the story of an ant crawling down the length of a magical rubber band. For every ant-length she walked, the rubber band was stretched a thousand ant-lengths. The question at hand was whether she would get to the end.

The answer is that she would eventually, provided the rubber band's growth didn't accelerate. This was because the rubber band behind the ant was being stretched too, pulling the ant along as she walked.

I immediately thought of this when a Youtube video reminded me of something I hadn't thought of in years: there were as many even numbers as natural numbers. I knew that this was the case because every

number can be doubled, but it still bothered me that I was still able to get an entirely different result when I generated a list of naturals and even numbers by adding one row at a time.

The scenario I concocted is as follows:

1. Create a list with two columns. Each row in Column A will house an iteration of x, where x is a natural number starting with 1. Column B will have a value of 2x, which is the equivalent of putting every even number down.
2. The existence of this higher number implies the existence of that number in column A. As such, each time a new even number is added to Column B, that number will be added to A, along with every number between the new even number and the numbers that are already on the list.

For example:

1 - 2

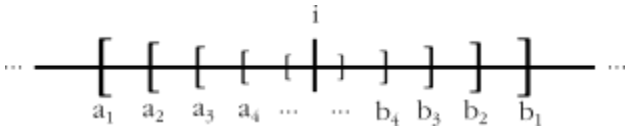
1 - 2
2 - 4
3
4

1 - 2
2 - 4
3 - 6
4
5
6

Note the growing divergence, but no new numbers are being filled in behind the leading number in the evens, so the total percentage of numbers covered by the evens never increases, even after an infinite number of steps. I've tried different ways of justifying it, but I just didn't understand why, when considering the total bijection of the evens and naturals all as complete mathematical objects, this divergence disappears?

This question intrigued me in the same way I imagine cosmologists view the Hubble Tension. I kept it in the back of my mind as I tried my best to understand Cantor's infinities. Along the way I encountered a lot of arguments for the uncountability of the real numbers which made me ask more questions.

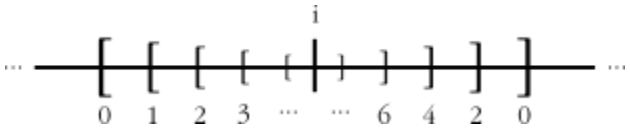
One of these questions involves the interval argument of the uncountability of the reals:




The arrangement of natural numbers on either side of i can converge to i, but never actually hit it. One runs out of naturals enumerating the intervals, so the naturals can't enumerate i or any points between the intervals.

I created a thought experiment based on this:

1. Fill in the left side with the natural numbers and the right side with the even numbers. Each opposing bracket counts as a bijection: 0 = > 0, 1 = > 2, and so on.

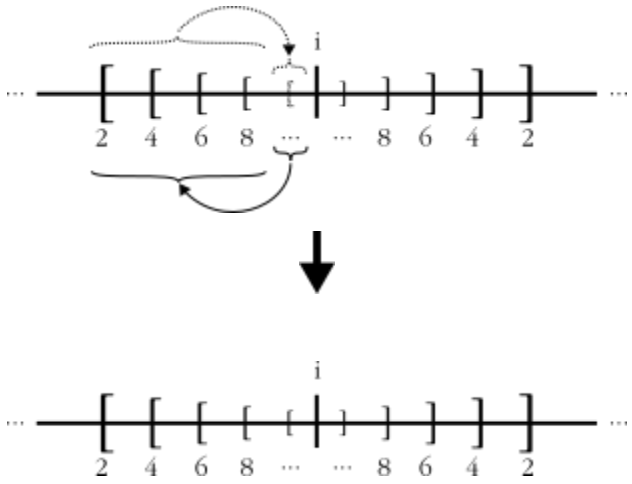



2. Remove every odd number from the naturals on the left, leaving gaps.

3. Rather than pull the evens away from i, we will shove them towards i. If we pulled the evens away from i, as one normally does in proofs by bijection, we would have no way to know what happens to the gaps beyond the ellipsis.

The first point at which there will be a contiguous series of occupied brackets without any gaps is when 0 reaches halfway between its old position and i. To push them beyond that would require more intervals than were there previously.

4. Swap the brackets that have even numbers attached to them with the gap brackets.

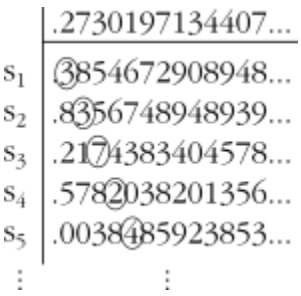



Now we have two seemingly identical lists of numbers. The presence of the gaps in the latter half of the left series forces us to conclude that while the numbers we can see match, halfway between 0 and i, numbers stop matching. This implies the following possibilities:

1. Somewhere past what we are physically capable of listing, the even numbers on the left are ordered in some specific way such that the entirety of the even numbers can only progress halfway to the evens on the right, even though both sides are ordered by increasing magnitude and should never stop matching.

2. The even numbers on the right only made it halfway to i. There are only half as many even numbers as naturals. Not every number can be doubled. Math breaks.
3. The list on the left only included half of the evens. This contradicts current theory.
4. The list on the right proceeded to a natural number that is twice as large as any natural number. This also contradicts current theory.

This, like any paradox, highlights the issue but provides no definitive resolution. Long before I thought of this, however, I focused mostly on Cantor's diagonal argument and the continuum. Here, one can have an infinite list of naturals paired with a random assortment of non-terminating decimals. One can then generate a new number that isn't in the old list by making sure each digit of the new number doesn't match one of the numbers on the list.




While studying this, I mostly found myself wondering why one would use a random assortment of decimals? After all, if Cantor used a random assortment of rational numbers to determine the rational numbers were uncountable, someone else would have created something like his proof of the countability of the rationals as a counterexample. In theory, if I were able to order the real numbers in such a way that they are one contiguous infinity then wouldn't the diagonal argument fail?

My immediate thought was to reject the premise. Cantor's theorem supersedes this. On the other hand, if the divergence between the evens and naturals doesn't matter, then why should Cantor's proof? If there was a way to show that counterexamples could be made with the same methods Cantor used to define the behavior of his system, then at the very least, the proofs of the countability of the rationals and integers would be suspect.

## Counting the real numbers

With the above in mind, I put myself to the task of finding a way to count the real numbers, eventually settling on the methods explained here.

The first step in the first method is to create the table below:

Table Zero

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0.01 | 0.001 | 0.0001 | 0.00001 | ... |
| 0.1 | -0.01 | -0.001 | -0.0001 | -0.00001 | ... |
| -0.1 | 0.02 | 0.002 | 0.0002 | 0.00002 | ... |
| 0.2 | -0.02 | -0.002 | -0.0002 | -0.00002 | ... |
| -0.2 | ... | ... | ... | ... | ... |
| 0.3 | 0.09 | 0.009 | 0.0009 | 0.00009 | ... |
| -0.3 | -0.09 | -0.009 | -0.0009 | -0.00009 | ... |
| 0.4 | ... | ... | ... | ... | ... |
| -0.4 | 0.99 | 0.999 | 0.9999 | 0.99999 | ... |
| 0.5 | -0.99 | -0.999 | -0.9999 | -0.99999 | ... |
| ... | ... | ... | ... | ... | |

According to my research, there are a countably infinite number of decimal places, but this only takes care of finite decimals between -1 and 1.

The next step is to split the above table into two:

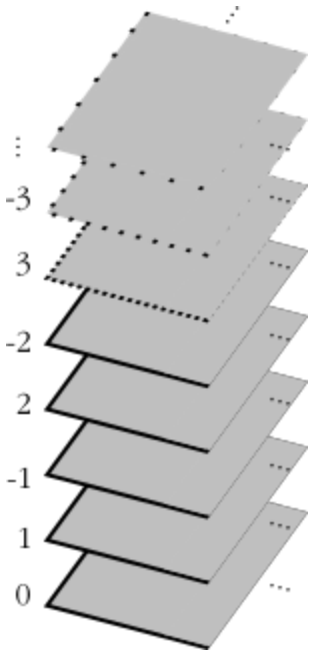
Table A

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0.01 | 0.001 | 0.0001 | 0.00001 | ... |
| 0.1 | 0.02 | 0.002 | 0.0002 | 0.00002 | ... |
| 0.2 | ... | ... | ... | ... | ... |
| 0.3 | 0.09 | 0.009 | 0.0009 | 0.00009 | ... |
| 0.4 | ... | ... | ... | ... | ... |
| 0.5 | 0.99 | 0.999 | 0.9999 | 0.99999 | ... |
| ... | ... | ... | ... | ... | ... |

Table B

| | | | | | |
|---|---|---|---|---|---|
| 0 | -0.01 | -0.001 | -0.0001 | -0.00001 | ... |
| -0.1 | -0.02 | -0.002 | -0.0002 | -0.00002 | ... |
| -0.2 | ... | ... | ... | ... | ... |
| -0.3 | -0.09 | -0.009 | -0.0009 | -0.00009 | ... |
| -0.4 | ... | ... | ... | ... | ... |
| -0.5 | -0.99 | -0.999 | -0.9999 | -0.99999 | ... |
| ... | ... | ... | ... | ... | ... |

Copies of Table A and B will be linked with the positive and negative integers respectively. This means that every entry in that table will have that integer added to or subtracted from it. We then stack the sheets vertically into a $\aleph_0$ x $\aleph_0$ x $\aleph_0$ sized structure.



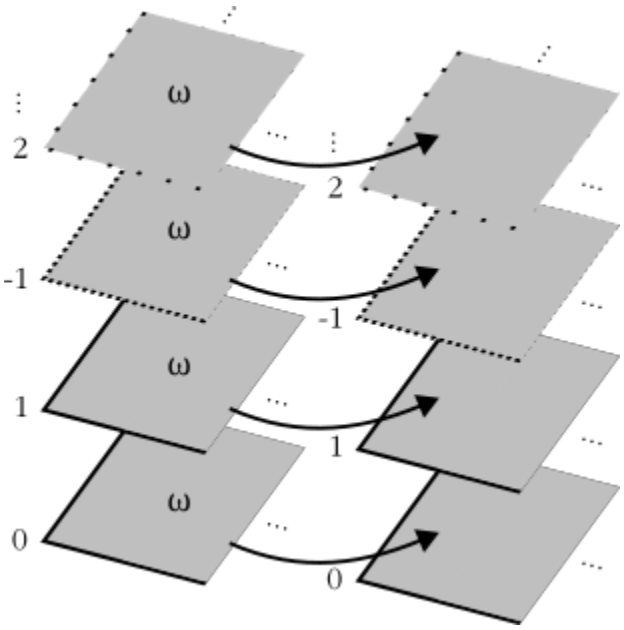

From here, we just need to make new tables for every decimal beyond the finite ones. As far as how to number these new decimal places, all one needs to do is retrofit the transfinite ordinal system to make a new group of tables starting with this one:

| | | | |
|---|---|---|---|
| $1 \times 10^{-\omega}$ | $1 \times 10^{-\omega-1}$ | $1 \times 10^{-\omega-2}$ | ... |
| $-1 \times 10^{-\omega}$ | $-1 \times 10^{-\omega-1}$ | $-1 \times 10^{-\omega-2}$ | ... |
| $2 \times 10^{-\omega}$ | $2 \times 10^{-\omega-1}$ | $2 \times 10^{-\omega-2}$ | ... |
| $-2 \times 10^{-\omega}$ | $-2 \times 10^{-\omega-1}$ | $-2 \times 10^{-\omega-2}$ | ... |
| $3 \times 10^{-\omega}$ | $3 \times 10^{-\omega-1}$ | $3 \times 10^{-\omega-2}$ | ... |
| $-3 \times 10^{-\omega}$ | $-3 \times 10^{-\omega-1}$ | $-3 \times 10^{-\omega-2}$ | ... |
| $4 \times 10^{-\omega}$ | $4 \times 10^{-\omega-1}$ | $4 \times 10^{-\omega-2}$ | ... |
| ... | ... | ... | ... |
| $9...999 \times 10^{-\omega}$ | $9...999 \times 10^{-\omega-1}$ | $9...999 \times 10^{-\omega-2}$ | ... |
| $-9...999 \times 10^{-\omega}$ | $-9...999 \times 10^{-\omega-1}$ | $-9...999 \times 10^{-\omega-2}$ | ... |

(For clarity's sake, I thought I'd note that the exponents listed there are $-\omega$, $-\omega-1$, and $-\omega-2$.)

We'll need to work in $\aleph_0 \times \aleph_0$ sized pieces, so we need a new table starting at $1 \times 10^{-2\omega}$ , $1 \times 10^{-3\omega}$ , and so on. From here, we just combine each entry with an integer as before.

We can't just stack this new group of sheets on top of the others, since there is no "top". That isn't really a problem because we can just shuffle any new tables into the old tables.




I wanted to be sure that I was hitting every real number, so I continued working under the assumption that I was wrong. To go beyond this, I started by counting at least some of the infinitesimals:

| | | | | | |
|---|---|---|---|---|---|
| 0ε | 0.01ε | 0.001ε | 0.0001ε | 0.00001ε | ... |
| 0.1ε | -0.01ε | -0.001ε | -0.0001ε | -0.00001ε | ... |
| -0.1ε | 0.02ε | 0.002ε | 0.0002ε | 0.00002ε | ... |
| 0.2ε | -0.02ε | -0.002ε | -0.0002ε | -0.00002ε | ... |
| -0.2ε | ... | ... | ... | ... | ... |
| 0.3ε | 0.09ε | 0.009ε | 0.0009ε | 0.00009ε | ... |
| -0.3ε | -0.09ε | -0.009ε | -0.0009ε | -0.00009ε | ... |

| | | | | | |
|---|---|---|---|---|---|
| 0.4ε | ... | ... | ... | ... | ... |
| -0.4ε | 0.99ε | 0.999ε | 0.9999ε | 0.99999ε | ... |
| 0.5ε | -0.99ε | -0.999ε | -0.9999ε | -0.99999ε | ... |
| ... | ... | ... | ... | ... | |

The rest of the tables follow the same pattern as the reals and are attached to every "integer" infinitesimal and then to each real number. The purpose of this is to form a justification for both the next series of tables as well as an alternate way to count the reals.

As for the next group of tables that we will be making, each entry consists of a nested table with three columns. The center part of each entry of this new table is a Cauchy sequence that converges on a real number with a ωth or greater decimal place. The other parts of each entry consist of two sequences that are defined by their last digit being one number above or below the corresponding entry in the base sequence. For example, here is the square root of 2:

| Lower bounds: | Base Sequence | Upper bounds: |
|---|---|---|
| $0+\varepsilon_n$ | 1 | $2-\varepsilon_n$ |
| $1.3+\varepsilon_n$ | 1.4 | $1.5-\varepsilon_n$ |
| $1.40+\varepsilon_n$ | 1.41 | $1.42-\varepsilon_n$ |
| $1.413+\varepsilon_n$ | 1.414 | $1.415-\varepsilon_n$ |
| $1.4141+\varepsilon_n$ | 1.4142 | $1.4143-\varepsilon_n$ |
| $1.4142+\varepsilon_n$ | 1.41421 | $1.41422-\varepsilon_n$ |
| $1.414212+\varepsilon_n$ | 1.414213 | $1.414214-\varepsilon_n$ |
| ... | ... | ... |

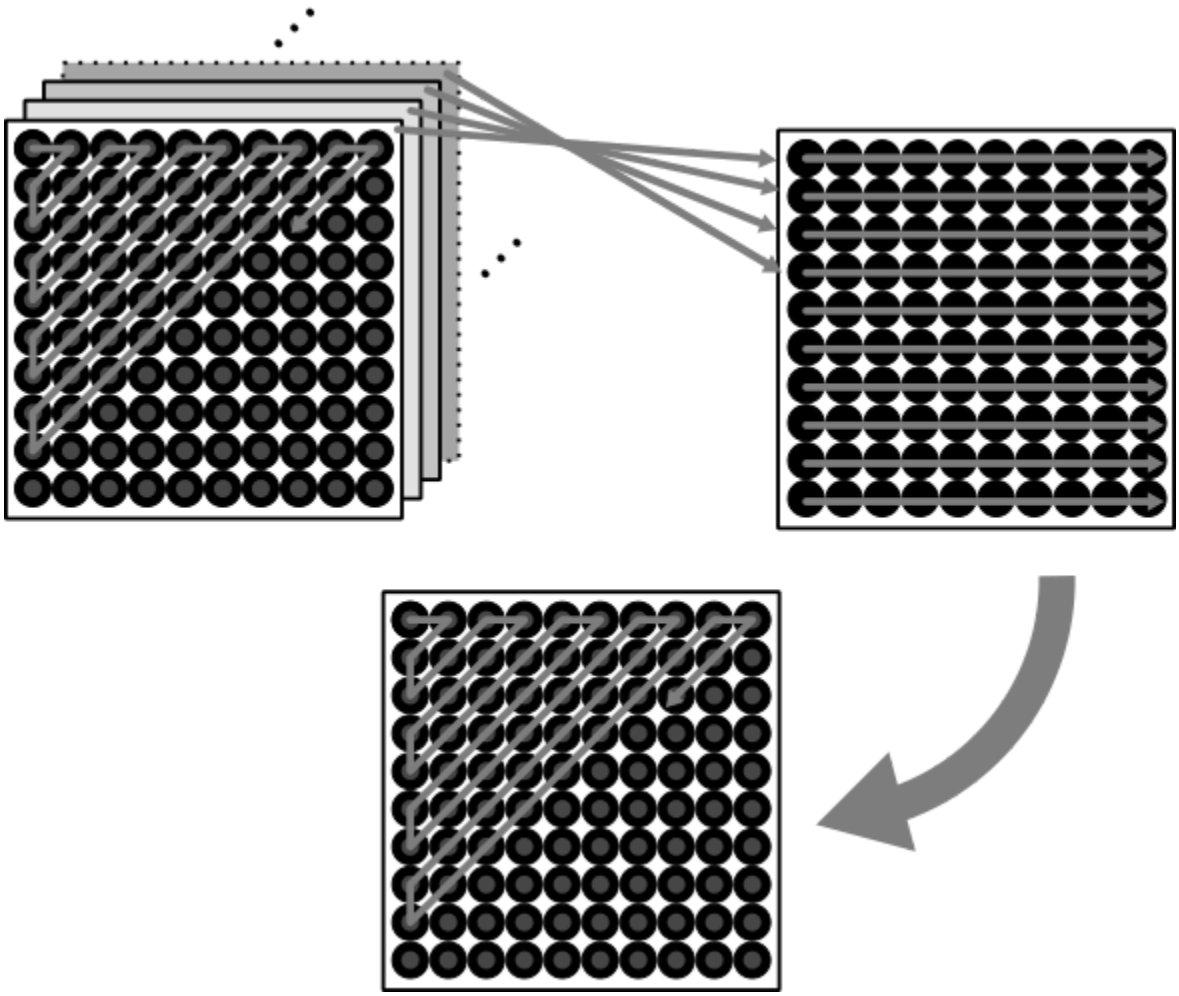
Note the infinitesimal notation added to the real numbers. This doesn't refer to a specific infinitesimal, but denotes that each entry listed there consists of yet another nested table that has every combination of the real component with every possible infinitesimal. In other words, the "n" stands for a series of infinitesimals that approach the real number component's closest neighbor in the direction of the base sequence. The infinitesimal part is to make sure we're converging more tightly than any Cauchy

sequence that only uses real numbers can converge. More tables are needed as the convergence continues into the $\omega$ decimal ranges.

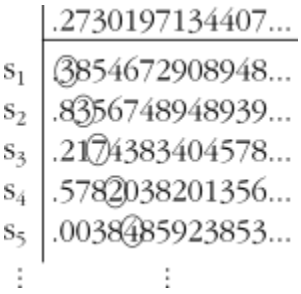
From here, we generate the sequences that will be used to define any last reals we're missing by going down the list, alternating between the lower and upper bounds columns to a level of precision that finally convinced me that I was able to list every real number.

We can then shuffle everything into the old stack, leaving us with the same-sized $\aleph_0 \times \aleph_0 \times \aleph_0$ stack.

Next, we can trace a path exactly like Cantor's proof of the rationals' countability through every single table we've created. This collapses each $\aleph_0 \times \aleph_0$ table into an $\aleph_0$ list. We can place each list onto a row of a new $\aleph_0 \times \aleph_0$ table, then draw a line through it to generate a final final $\aleph_0$ sized list.



Now we've generated a list that we can use to test the diagonal proof. For the sake of argument, let's say we randomize the list so that it comes out in this order:



We'll now try to construct this number one digit at a time.

So, .2 does not match the first digit of the first number, but 0.2 does exist on Table Zero. .27 doesn't match the second entry, but 0.27 is also on Table Zero. There is never a point where we can make a number with a digit that doesn't match something on Table Zero or any of its ω+ auxiliary tables.

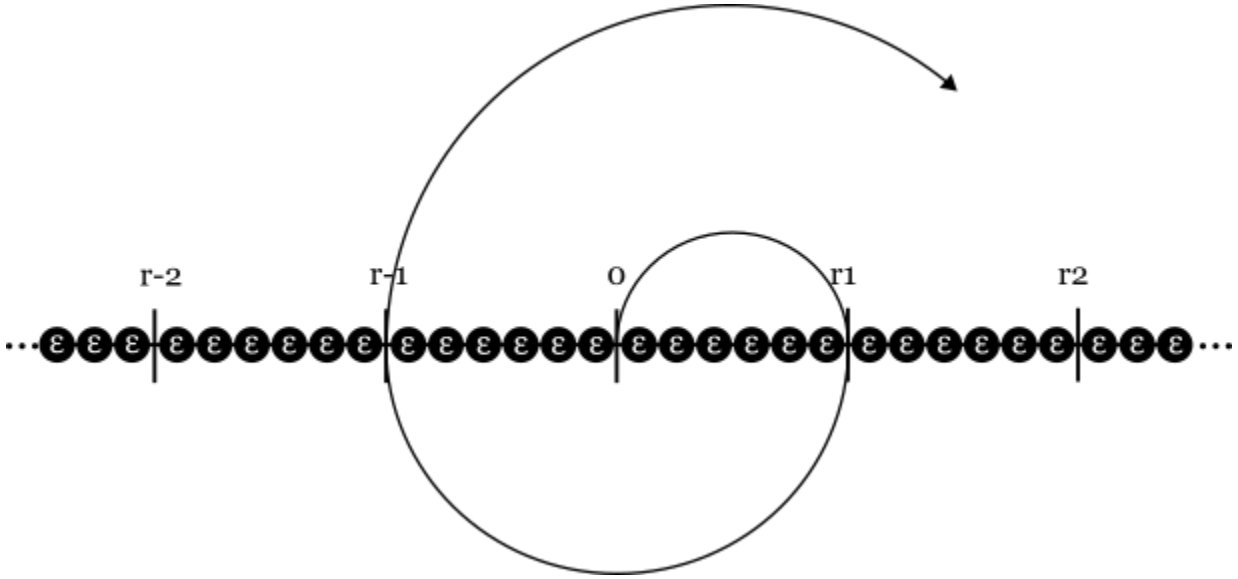
Of course, we're not really supposed to consider this one digit at a time, so let us consider the number as it would appear in its entirety. I find it easiest to visualize this number as it would appear on the hyperreal line. There will be a digit where, if we were to go beyond that, would land us in the infinitesimals. We could fill in a number that doesn't match whatever digit of whatever number we happen to be looking at in our list, but the other nine possibilities would be listed elsewhere.

Any attempt to add an integer will also fail, as we've listed those as well.

I've been unable to find a way to get the diagonal argument to work on this list.

The above method can also be adapted to count the power set of the naturals directly. One uses the first table for the empty set, the naturals, and natural number combinations with two elements. The dimensions of the table go up from there, eventually reaching infinite dimensions. Everything can still be broken down to $\aleph_0 \times \aleph_0$ tables, however, so they can all be shuffled and collapsed.

There are other ways to count the reals. Remember that we counted the infinitesimals as well, or at least a lot of them. Assuming that we can't count them all, then our inability to count them applies equally across the entire number line. This means there will be equidistant gaps between what we can and can't count. We can use this information to construct enough of the hyperreal number line that we can generate a spiral that hits every real number precisely.




The natural objection is that Cantor's theorem invalidates all of this. The fact remains that we don't physically lose our ability to draw a line after a certain point. The real numbers are being paired with *something* that can't be reached with natural numbers alone, but our definition of natural numbers dictates that those numbers *must* be natural, since each time the line crosses an element on the table or number line, that number at that position is equal to the sum of a finite number plus 1.

With nowhere else to turn, I tried to see if I could create a counterexample to Cantor's theorem, just in case there really was just one infinite cardinality. After some thought and research, I came up with three scenarios that highlight more problems with Cantor's methodology.

All three scenarios below are based on the following:

1. All infinite subsets of the naturals have the same cardinality as the naturals. All power sets of infinite subsets of the naturals have the same cardinality as the powerset of the entirety of the naturals.
2. Any countably infinite subset of the naturals should be able to enumerate any other countably infinite set. Likewise, any infinite power set can count any other infinite power set.
3. We know that the naturals can't have a 1:1 bijection with their powerset because of a "landmark" set that we can track. This is the set of all natural numbers that biject with a subset of P(ℕ) that does not contain the paired number (called selfless numbers, or set B in the Wikipedia article) within its elements. For the purposes of this draft, I'm keeping the explanation short. The full article is here: https://en.wikipedia.org/wiki/Cantor%27s_theorem

Scenario 1:

1. First attempt to enumerate the power set of the evens (P(E)) with only the even numbers.
2. Set B of this particular powerset just has even numbers, so we can use odd numbers to enumerate it and any remaining sets without contradicting Cantor's theorem.
3. Now attempt to pair the powerset of the evens with the powerset of the naturals while keeping the bijections we made "handcuffed" to one another.
4. The bijection of the evens will follow its power set to biject with the powerset of the naturals, showing that there is a 1:1 bijection between ℕ and P(ℕ)

I used to think this meant that there really was only one infinity. However, consider that while P(ℕ) contains the combined set of the selfless evens, it also contains the combined set of the selfless evens along with the number 1 as an additional selfless number, called B1. It also contains set B2, which is just B1 combined with 3, then 5, and so on.

| Evens | P(E) | P(ℕ) | |
|---|---|---|---|
| 2 | { } | { } | |
| 4 | 2, 4, 138 | 0, 4 | |
| 6 | 12, 8 | 9, 390, 2 | |
| 8 | 440, 14, 2, 281710 | 95, 3889, 3940... | |
| 10 | 4724, 30, 1790... | 5, 6, 7, 9, 25 | |
| ... | ... | ... | |
| Pn-2 | 108, 434, 33 | Nn-2 | |
| Pn-1 | 23018 | Nn-1 | |
| Pn | 690, 3374, 7490... | Nn | |
| 1 | 2, 4, 6, 8... | 0, 1, 2, 3, 4, 5... | SET B |
| **3** | **?** | **1, 0, 2, 4, 6, 8...** | **SET B1** |
| **5** | **?** | **1, 3, 0, 2, 4, 6...** | **SET B2** |
| ... | ... | ... | ... |

I then naively concluded that you can enumerate P(E) with ℕ, but you can't enumerate the entirety of P(ℕ) with ℕ. The larger point that I was missing at the time was that there are many different variations of a set B in both finite and infinite subsets of ℕ, and not every subset B is in other subsets of ℕ.

Within P(ℕ), for every number n, there is a version of set B that only differs because n. In other words, n can enumerate that set B, another selfless element, or a selfish element. There is also a copy of set B that is minus n and any number of extra elements removed. So two numbers can possibly be paired with it, but each of those numbers can only be paired with either this or the numbers in the other categories I mentioned.

It follows that less selective subsets (infinite or otherwise) of ℕ will have more variations of set B than more selective ones. For example, the powerset of all multiples of 8 will have set B variations for multiples of 8, 16, 32 and so on, but not for every multiple of 4 or 2.

So, rather than just selfless and selfish numbers, we find that numbers can biject only with one, and only one, of the following categories (DRAFT NOTE: I'm sure I'm missing a lot. Please let me know if you can think of any other categories):

- The empty set
- The singleton subset of P(N) that matches n exactly.
- Any subset of P(N) that does not contain n.
- Any non-set B subset of P(N) that contains n.
- Any incomplete infinite variant of set B that does not contain n.
- A set of all *selfish* numbers (set C)
- Any of the complete and incomplete variations of set C and subset C.

For example, if we were to pair n with the empty set, then n+1 with the singleton subset that matches n. That means n+1 has none of its possible pairings occupied, meaning it's a selfless number. As such, it cannot be in set B or any of the other categories.

In contrast to the above, we find that selfless numbers cannot biject with any of these:

- Set B
- Any variant of Set B that still includes n.

Through some clever arrangement of the categories that each natural can and cannot inhabit, I believe one can track the sizes of sets between $\mathbb{N}$ and P($\mathbb{N}$) at a granular level. I eventually abandoned this as I created an alternate construction of numbers that rendered this moot.

Scenario 2:

1. Enumerate P(N) with natural rational numbers (i.e. 1/1, 2/1, 3/1, etc.)
2. When set B is encountered, pair a non-natural rational number with it.
3. Use the rest of the non-natural rationals to enumerate any remaining sets.
4. If there are any subsets left over, refer back to Cantor's original proof of the countability of the rationals. It doubles as a lookup table that matches a natural number to every one of the rationals, including the rationals's natural number equivalents. We just need to match every rational number on our attempted bijection with the natural number that intersects its position on the table, then just swap out every bijected rational for the natural number at that position.
5. If it turns out that there are still subsets left over, all one needs to do is swap out the naturals in this new bijection for their rational number equivalent and repeat steps 2-4.

There are two conclusions we can draw from this:

1. One can enumerate anything with the naturals even if it should be logically impossible. There are as many natural numbers as real numbers.
2. The above method doesn't actually work. There must be more rational numbers than natural ones.

Scenario 3:

1. Assume that there is a list of empty sets $2^{\aleph_0}$ elements long called L. We're just concerned with cardinality so it shouldn't matter how each element is labeled.
2. Attempt a bijection with consecutive naturals until we get to where set B would normally be located. Before this point, there is a natural number n.

The existence of this bifurcation point leaves us with the following questions:

1. What is preventing us from adding 1 to n to get a natural number to pair with set B?
2. Would we be able to reach set B if we only used the even numbers to get here? If so, can we then use the odd numbers to enumerate set B and beyond? Why can we not repeat this to enumerate the entirety of L?
3. What is preventing a person from multiplying n times 2?
4. Where is 2n on the natural number line?
5. N has a prime factoring, so it should be accessible via arithmetic, but current number theory says that only the smallest naturals are accessible via arithmetic. Why?
6. If n is an even number, then n should appear halfway between it and zero in a bijection between the evens and the naturals. What numbers are paired with the second half of the naturals?

We can dismiss all three of these scenarios as either paradoxes or too ambiguous to provide any useful information, but that doesn't change the fact that it appears that we can endlessly cascade between the naturals and the rationals to count anything. Those variations of set B still exist in P(N). There is still a bifurcation point between a list of the naturals and L that should be paired with a natural number whether or not it is logically consistent to do so.

## Recounting the rationals

In order to reconcile these conflicting facts, I needed a way to examine the entire natural number line at once, as well as strictly track the locations of every number. I've gone through a few ways to embed the natural numbers in a finite space, including repurposing the interval argument, placing random numbers on a line, and John Conway's base 13 function, but I eventually settled on the below technique. This is because it starts with a sequentially written list of natural numbers and we can use this fact to explore a paradox.

To make a completely explorable natural number line:

1. Take the list of natural numbers and split them into evens and odds

...8 6 4 2 0 1 3 5 7 9....

2. Take the two lists and hold them an uncountably infinite distance away from each other and rotate them 180 degrees.

0 2 4 6 8... ...9 7 5 3 1

3. Push the two lists together, then move back an uncountably infinite distance away. We'll see the whole list as a line composed of an $\aleph_0$ number of bits.

4. Since it is just two lists of numbers and not a continuous line, we can just pretend there's a zero-dimensional point in the middle of each number that we can expand into half-centimeter wide circles, stopping when the edges touch. What happens at this scale will happen at every scale, so the line has been stylized like below. The black circles are evens and the white circles are odds.

5. Here we will swap around each entry on the list so they alternate between even and odd in ascending order, keeping zero and 1 at their positions, once again making the list into one continuous infinity. It is important that we swap the numbers here so we can make sure no new numbers are being pulled out of nowhere.

So now we have a list of the natural numbers where every single number is accounted for. If we remove a number, there will be a gap that needs to be filled in by shifting a number from the left or right to it. Since there are endpoints, that gap will eventually get to the zero or 1 with nowhere else to go.

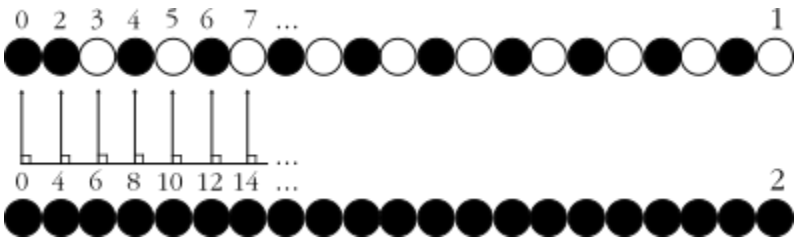
And much like the third scenario in the last section, the natural numbers suddenly have a "last" value. Any concerns about this are well founded, but questions of magnitude are typically ignored in discussions of set cardinality, so we will keep with conventions for now.

One can still argue that this new arrangement will just flow past the 1 and off to some other place, but remember that we just swapped the same amount of numbers around. Making the list go past our bounds would require adding numbers that weren't on the natural number line before, and thus would be larger than any natural number.

What's important is that we're keeping track of a countably infinite number of objects, not the magnitude of the individual numbers in the set.

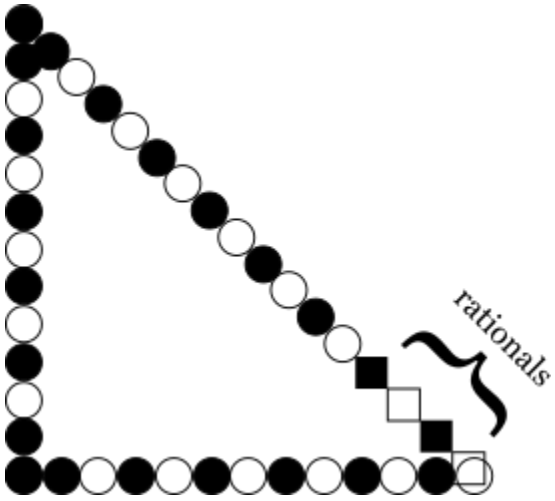
Anyone still uncomfortable with this can choose any other method to embed a countably infinite number of points in a finite space. It does not affect the behavior of equidistant points on a line.

Continuing on, since the elements in the list were evenly spaced, we can use this to investigate the list geometrically. What this means is that every 1:1 bijection will be perpendicular to the direction of the list.




We can use this property to explore the paradox I mentioned.

Below we have three lists and we're attempting to build a right triangle. We need to add a few rational numbers to the end in order to get it to work:




We will now try to make a new hypotenuse by doing the following steps.

First, we start a new list of numbers:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ...

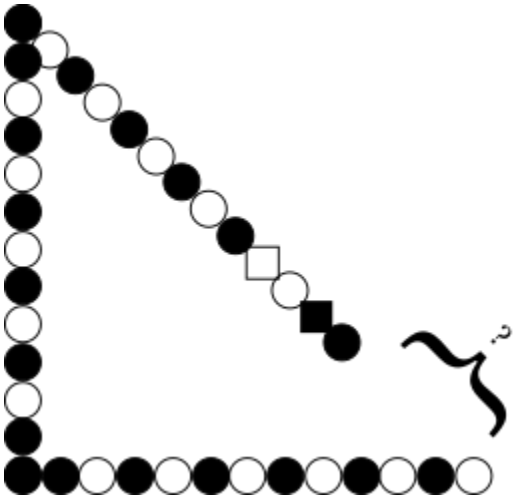
From here, we shuffle the rational section of the old hypotenuse into the new list.

$0, q_1, 1, q_2, 2, q_3, 3, q_4, 4, q_5, 5, q_6, 6, ...$

We then re-embed the new list in a finite space as before, keeping the circles and diamonds the same half-centimeter in size:

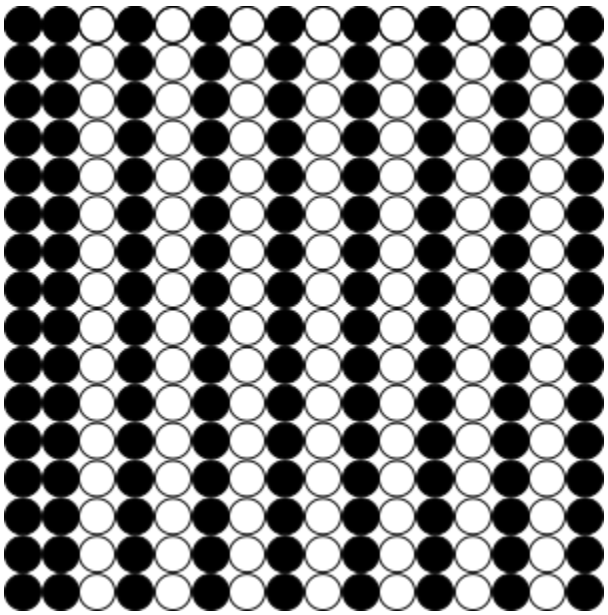
We will once again try to make a right triangle with this new hypotenuse. Since we know that this new list will have a cardinality of $\aleph_0$, all bijections will be perpendicular to the direction of the list. As such, because every element of the new list must have a 1:1 bijection with the opposite and adjacent lines of the other sides of the triangle, all three lines must be the same length.

As a result, we can no longer make a right triangle with this new hypotenuse:



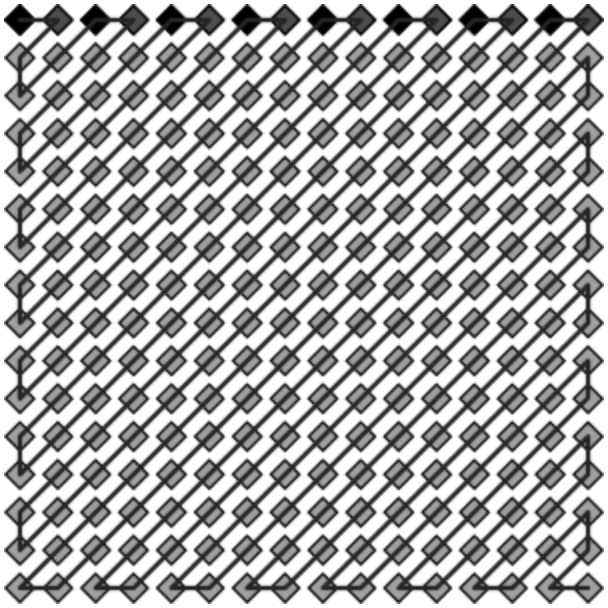
It's interesting to ponder, but we won't find any definitive answers here, either.

To get something useful out of this, let's construct the entirety of Cantor's proof of the countability of the rational numbers. Since we know that the rationals are made by combining pairs of naturals, we can create an $\aleph_0$ sized stack of number lines, effectively making an enormous grid.



From here, we can replace every natural number there with a rational number. The rational naturals run across the first row of the chart.

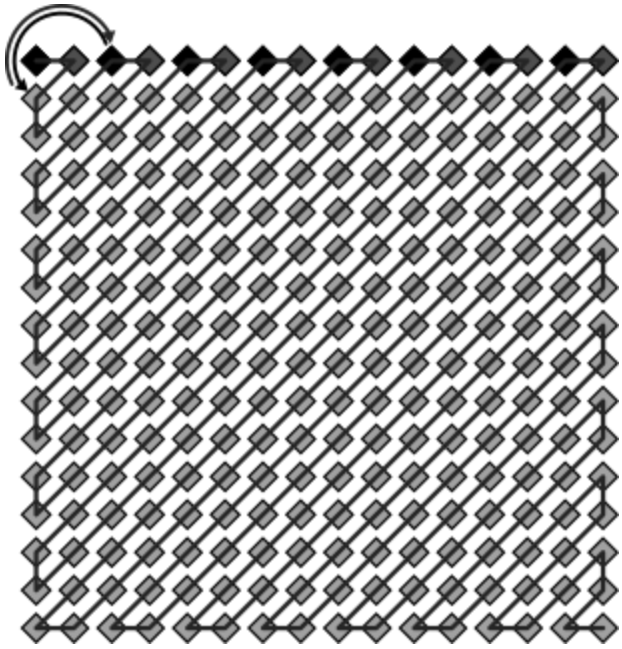
We can also generate the line that runs through the chart:



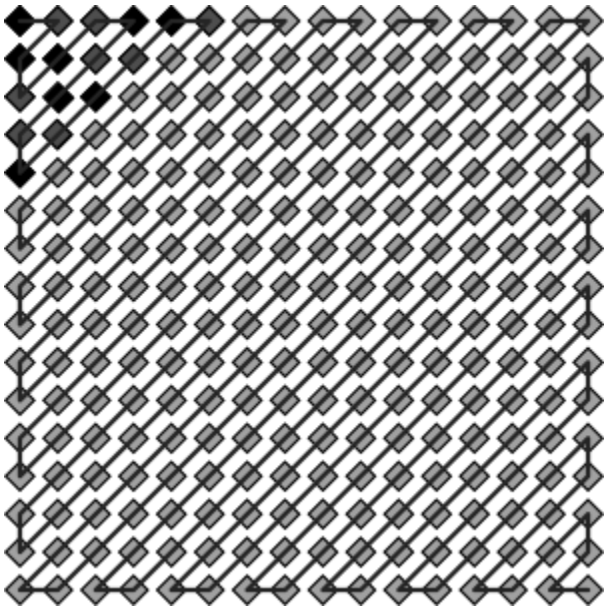
As a reminder, each time the line hits a rational number, that counts as a bijection with a natural number, which I will be calling the position number. Now we are able to consider the proof in its entirety.

We will now trace down the line that runs through the table. Each time we hit a position on the table, compare that number's value with the position number attached to the line.

If the position number on the line doesn't match the number on the table, *swap* out the corresponding number on the table for a rational natural that matches the position number on the line. The swapping is necessary to make sure there's a full accounting of every entry on the table and to make sure that no rationals are pushed off the table somehow.



The end result is that the naturals are pushed up to the top left of the table while non-naturals are pushed onto the rest of the top row of the table:



The table is now divided into two regions. Region A has every natural rational number from the table paired with a matching natural number on the line. Region B has everything else on the table paired on the line with a number of some sort.

We know the value of the numbers on the line haven't changed. We also know that the values on the table were just swapped around, so nothing's changed there. The natural rationals and the line naturals are also supposed to be ordered by magnitude and should never stop matching. As a result, there are only three conclusions that I can see:

- The set of rational natural numbers is smaller than the set of natural numbers. This is impossible because the naturals were used to generate the rational natural numbers.
- The numbers on the line stop matching the naturals on the table somewhere but the numbers on the line are still naturals. In other words, there exist natural numbers that don't match any natural number but are still within the set of natural numbers. This is a contradiction.
- $\mathbb{N} < \mathbb{Q}$. And since we can just as easily apply this method to the "bijection" I created for the reals, that means that $\mathbb{Q} < \mathbb{R}$. It also follows that this can be used to show that there are half as many naturals as integers, and half as many even numbers as natural numbers. Math breaks again.

One may say that the naturals on the table are ordered sequentially while the naturals on the line are just some other arrangement of the naturals that can be used to cover the entire thing.

If it's a problem with the ordering, then can we, through swapping alone, order the natural rationals in region A in such a way that they occupy the entirety of the line?

This isn't an ordering problem. The problem is that the concept of order types breaks down when we observe an infinite set in its entirety.

It's now time to reconsider the bijection between the evens and the naturals. As far as I remember, the notion that there are as many even numbers as natural ones came about as one of Euler's margin notes that he really didn't think too much about. Other mathematicians worked off this assumption because it isn't really that much of a leap. The mapping of the evens to naturals is also a mapping of every number to twice that number. Every number can be doubled. It's about the easiest proof in the world to accept because this needs to be true for math to work.

Nevertheless, we still need to verify it, however illogical the pursuit may seem. If it isn't true, then we need to figure out what is. So here is the first part of the bijection of the evens to the naturals– about as many as can fit in Euler's margin notes:

2, 4, 6, 8, 10...
1, 2, 3, 4,  5...

Observing the above, I couldn't help but notice a difference in our physically observable dataset and the totality of natural numbers. In our dataset, in order for there to be a bijection between 3, 4, and 5, one must invoke the existence of numbers that aren't in the visible dataset. In other words, we are using an open set to make inferences about the nature of the natural numbers- a set that is closed under addition and multiplication. If we were to study this issue as a closed system, it would look like this:

2, 4, 6, 8, 10, ?, ?,  ?,  ?,  ?...
1, 2, 3, 4,  5, 6, 7, 8, 9, 10...

I know this is pointing out something elementary. The argument is that there is not only a quantitative difference between the finite and the infinite, but a qualitative one as well. This is a concept that we can investigate by first bijecting the natural numbers with themselves:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10...
1, 2, 3, 4, 5, 6, 7, 8, 9, 10...

We now will attempt to recreate the bijection between the evens and the naturals, but we will try to do so through swapping. This ensures that both sets remain exactly the same size and don't start pulling in numbers from outside the set.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10...
**2**, 1, 3, 4, 5, 6, 7, 8, 9, 10...
**2, 4**, 1, 3, 5, 6, 7, 8, 9, 10...
**2, 4, 6**, 1, 3, 5, 7, 8, 9, 10...
**2, 4, 6, 8**, 1, 3, 5, 7, 9, 10...
**2, 4, 6, 8, 10**,  1, 3, 5, 7, 9...
^ ^ ^ ^  ^  ^ ^  ^ ^ ^
1, 2, 3,  4,  5,  6, 7, 8, 9, 10...

The evidence that we can directly observe trends towards there only being half as many even numbers as naturals. The only way for the evens to get past this point is to start swapping with numbers outside of the set of natural numbers.  I'm forced to conclude from this and every other experiment that Euler's observation that there were as many even numbers as naturals only appears to show that every number can be doubled, not how finite that particular number would be.

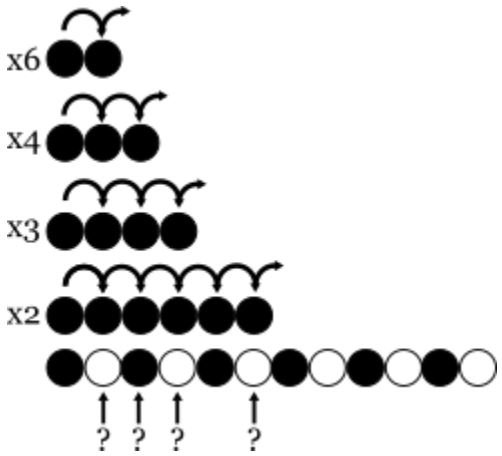
This calls the closure property of the naturals into question. Falsifying an axiom has consequences beyond having to rework a theory or two, however. It forces us to question what we even mean when we think of a number as finite.

## Constructing the supranaturals

My first eventful attempt to reconcile all of these seemingly conflicting facts started when I attempted to force the geometric list construction of the natural number line to exhibit closure in whatever way was internally consistent, just to see what would happen. As such, I returned to the bijection between the evens and the naturals. We know that there are an infinite number of even numbers, so the set of even numbers can be thought of as a guide to the number of hops the number 2 needs to make to "jump" past the naturals. Likewise, multiples of 3, 4, etc. will chart smaller infinities.

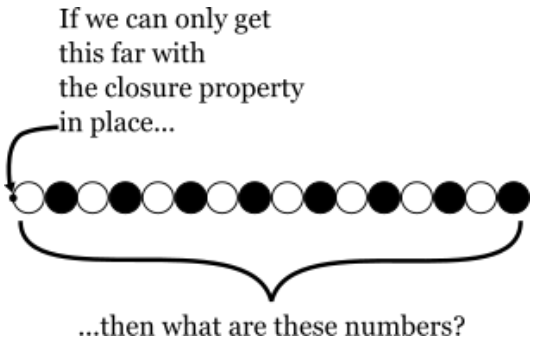
In other words, we would need an infinite number of hops to get past even the smallest infinite set. But since each hop matches a natural number, then there are numbers in this model that would violate the current definition of what it means to be finite.

More to the point, what about the numbers on the right half of the list? Multiplying any of them by 2 will allow the result to cross over into "transfinite" territory somehow.



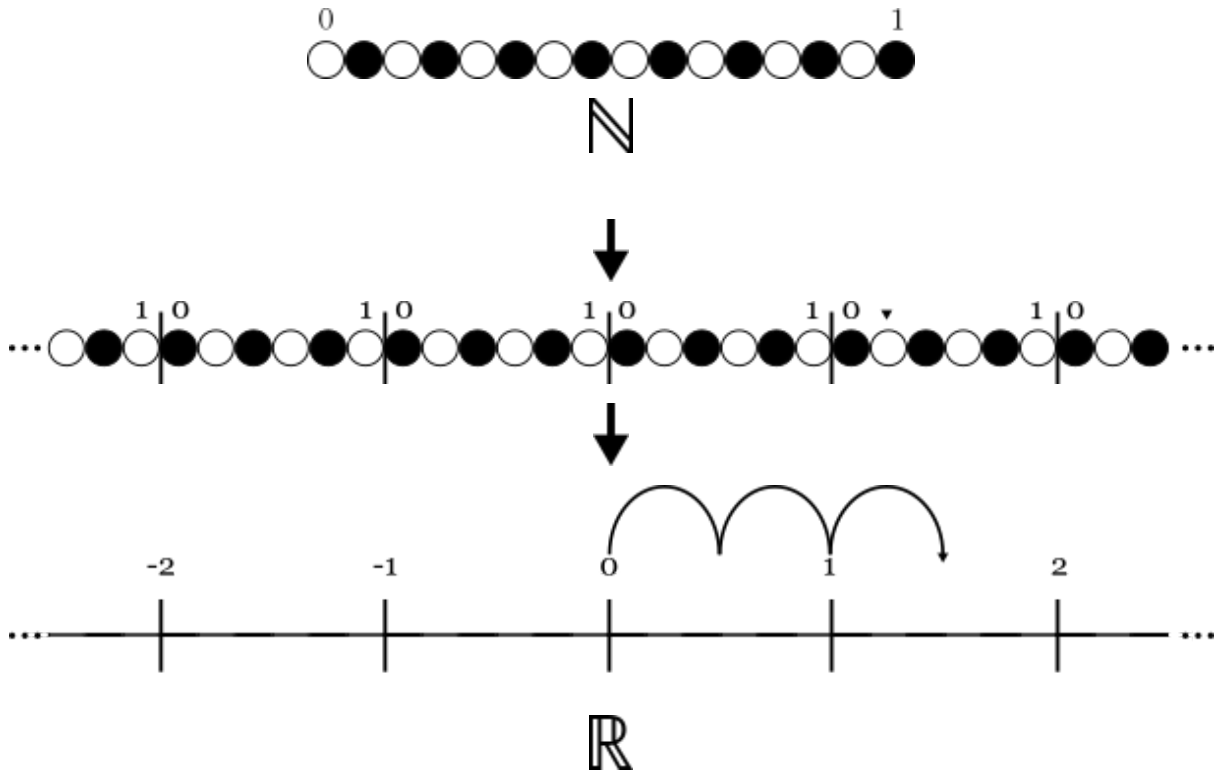

Keeping all of that in mind, let's consider the traditional construction of the natural numbers. Multiplying any two finite numbers will always result in a finite number. That result, no matter how large, will always have an $\aleph_0$ amount of numbers ahead of it. Nothing but some kind of infinite iteration will allow a person to get past that point, but then the result just goes off to the limit, which isn't really helpful in understanding what goes on when considering the naturals as a set.

So with the closure property in place, both models only allow the very smallest numbers to be accessed through arithmetic. Between those numbers and the "end" of the naturals are numbers that seem to defy what it means to be finite.



And how can a person get to those numbers? They should have a unique prime factoring after all. This vexed me for a long time until I realized that you can:

1. Truncate an infinite amount of number lines end to end.
2. Cut away all of the circles connecting the numbers in the list, except for a line running through the diameter of each circle, forming a line.
3. Remove the number labels and just use a coordinate system that is defined by a location's exact distance from a center point (marked as 0).
4. Relabel it as $\mathbb{R}$.




And we are now able to spring past infinite quantities of real numbers with finite arithmetic.

So what is the overarching problem with this and every other part of this exploration? The best analogy I can think of has to do with black holes. For a while, the mathematics describing black holes had two

singularities: one at the black hole's surface and one at its center. It eventually took the Eddington-Finkelstein coordinate system to eliminate the singularity at the surface.

In other words, the core issue appears to be that Cantor's model of infinites produces singularities at its boundaries. If that's the case, we just need a different governing philosophy when constructing and manipulating infinite sets. If successful, we will have a model of numbers that contains no conflicts. Unfortunately, we can't do this without breaking down some psychological barriers.

The first barrier that must be brought down is that our idea of an infinite set as an ascending staircase of elements leading to an unreachable boundlessness is the only valid one. The second barrier is the notion that there must be clear borders between the finite and the infinite. Lastly, that the closure property is not necessary to have a functioning construction of numbers, even if this causes trouble for mathematics in the short term.

With regard to what we will use to replace the closure axiom, my best description is something like an axiom of phenomenological co-emergence, but I don't have the necessary expertise to construct something formally. I can only speak of it in philosophical terms.

Basically, it means that everything is defined relative to everything else and nothing has a nature of its own. Here is a classic example:  a tree can't be defined without the water, soil and sunlight that sustain it. We couldn't define the sunlight without the sun, and we couldn't define the tree, sun or the earth without the totality of space and matter between and around them, beyond the observable universe, spanning infinitely into the past and future, the concepts of what is possible and impossible, and beyond. Of equal importance is that the tree and the totality of the context surrounding it can't be described without also describing an observer. Finally, any description of that tree, and by extension, everything, is not a description of the observer or observed so much as it is a rough description of the experience of observation.

Applying this philosophy to the even numbers, they cannot be treated as existing in and of themselves. When we consider the evens, we also consider the implied existence of the odds, even if we're not specifically listing them. We do this because "even-ness" would be meaningless in the absence of "odd-ness".

For example, if there were a hyperdimensional alien species that, rather than existing in one universe at a time, straddled two realities simultaneously, their version of the number 1 would look something like this:

:

And it would blow their minds if they found out that their versions of ½, 1 ½, etc. are our odd numbers.

In other words, we will not be considering numbers as axiomatic logical objects, each in their own vacuum. They will be considered as they are relative to one another.

Finally, every number has a specific magnitude, and this magnitude is relevant even in discussions of cardinality. As a result, everything must be strictly tracked.

Abiding by these principles, I found a way to construct numbers in such a way that resolves every single issue I've encountered. I couldn't just stipulate an infinity into being and put some labels on it, though. I needed a mechanism to construct this infinite set that doesn't involve the natural numbers. There are a few ways to do this, like counting how many times the right side of an area can be divided in halves/thirds/etc. These would all produce infinities of different sizes under this system, but the behavior we will be establishing will be consistent across all conceptions of infinite sets, so it isn't that important how we create a stack of an infinite number of '1's.

For this construction, we will start with a one-dimensional line segment. This segment will be cut into points so small that cutting any more would result in three cuts in a row:

The bounded infinite set above is not the number line, however. This is a number with a specific (though admittedly poorly defined) infinite magnitude. Numbers like this are called suprafinite, as

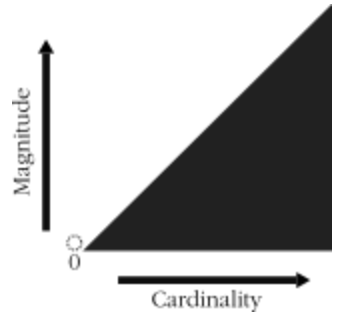
opposed to classically finite. This distinction is just to aid in discussing these numbers– there is no special border between classically finite and suprafinite numbers.

The next step in this construction is to pair it with 0:

We can then make a copy of this unit, subtract 1 point and put that unit next to the point.

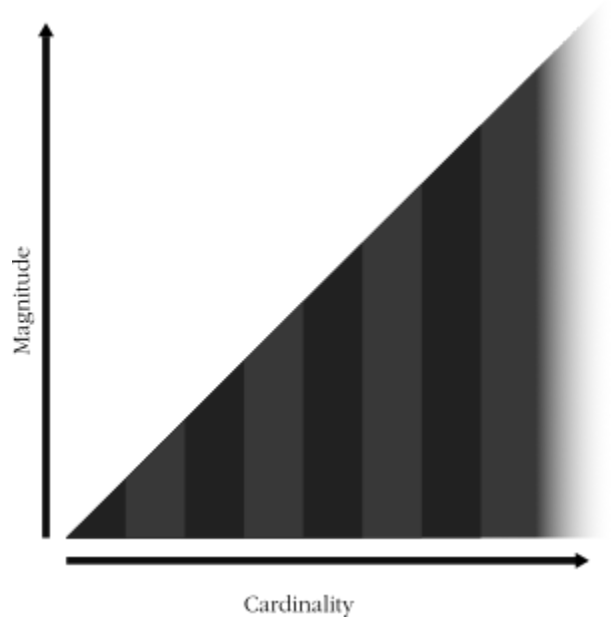
In between these two units, we'll put another unit copy and subtract 2 points.

This pattern continues until we get this shape:




In keeping with the co-emergence property, a number that is equal to the unit we created +1 must exist, since that is where the pattern heads. We can keep doing this endlessly, making a seemingly boundless line of numbers where the magnitude of a particular number will always equal the cardinality of all the numbers up to that point.

We'll call this set the supranaturals or $\mathbb{S}$.



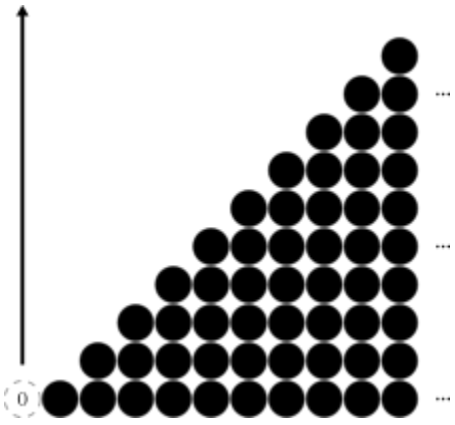

As noted before, there is no line between the classically finite and suprafinite in this construction. Relative to the unit we made, smaller numbers lose a bit of "infiniteness" as they trend towards

classically finite numbers, with 1 being the only truly finite number. Likewise, numbers larger than the unit are more suprafinite. This is true of any number, however. The unit infinity that was constructed at the beginning of this exercise isn't special in that regard.

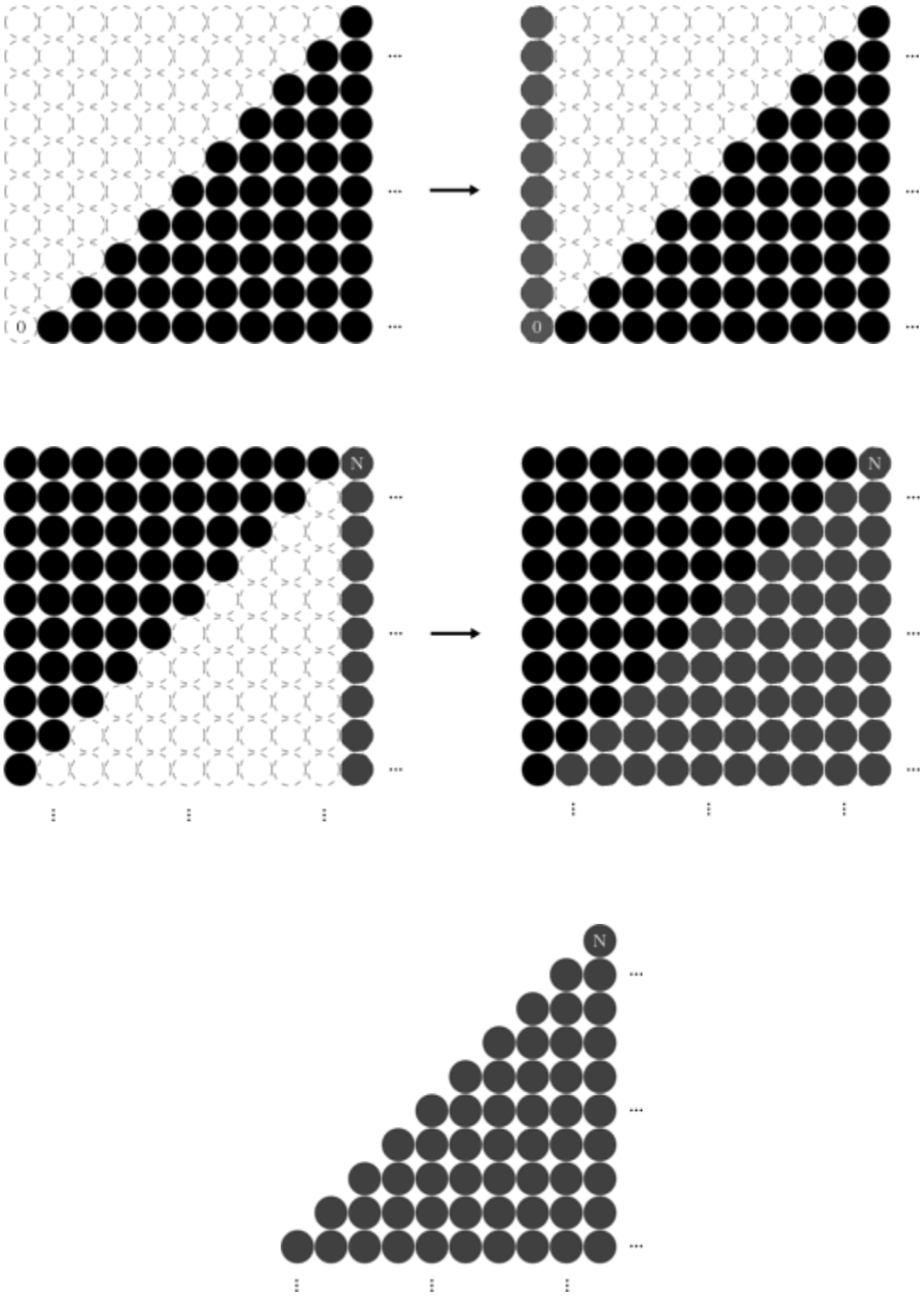
With regard to numbers with special qualities, the question as to whether there is a number analogous to $\aleph_0$. In Cantor's system, $\aleph_0$ is a number that is equal to the total number of natural numbers, but it is beyond the reach of the natural numbers themselves. We can stipulate that there is a number that exists like that in this system, but remember that magnitude and cardinality have a 1:1 ratio, and numbers can be of infinite size. Because of this, the "$\aleph_0$" of this system should be on this number line.

We can test this in a couple of ways. For this first thought experiment, we will go to the very beginning of the supranatural line. It trails off to some ill-defined boundlessness to a theoretical infinite number with a value N.

Examining our number line, there isn't a clear way to get to N by just travelling to the right, and it is not acceptable to just stipulate that we are at N. We can only work with what is currently visible to us. However large this number is, we do know that the magnitude will be equal to at least the number of gaps in the zero column, as we know that we can subtract N from itself to equal zero.

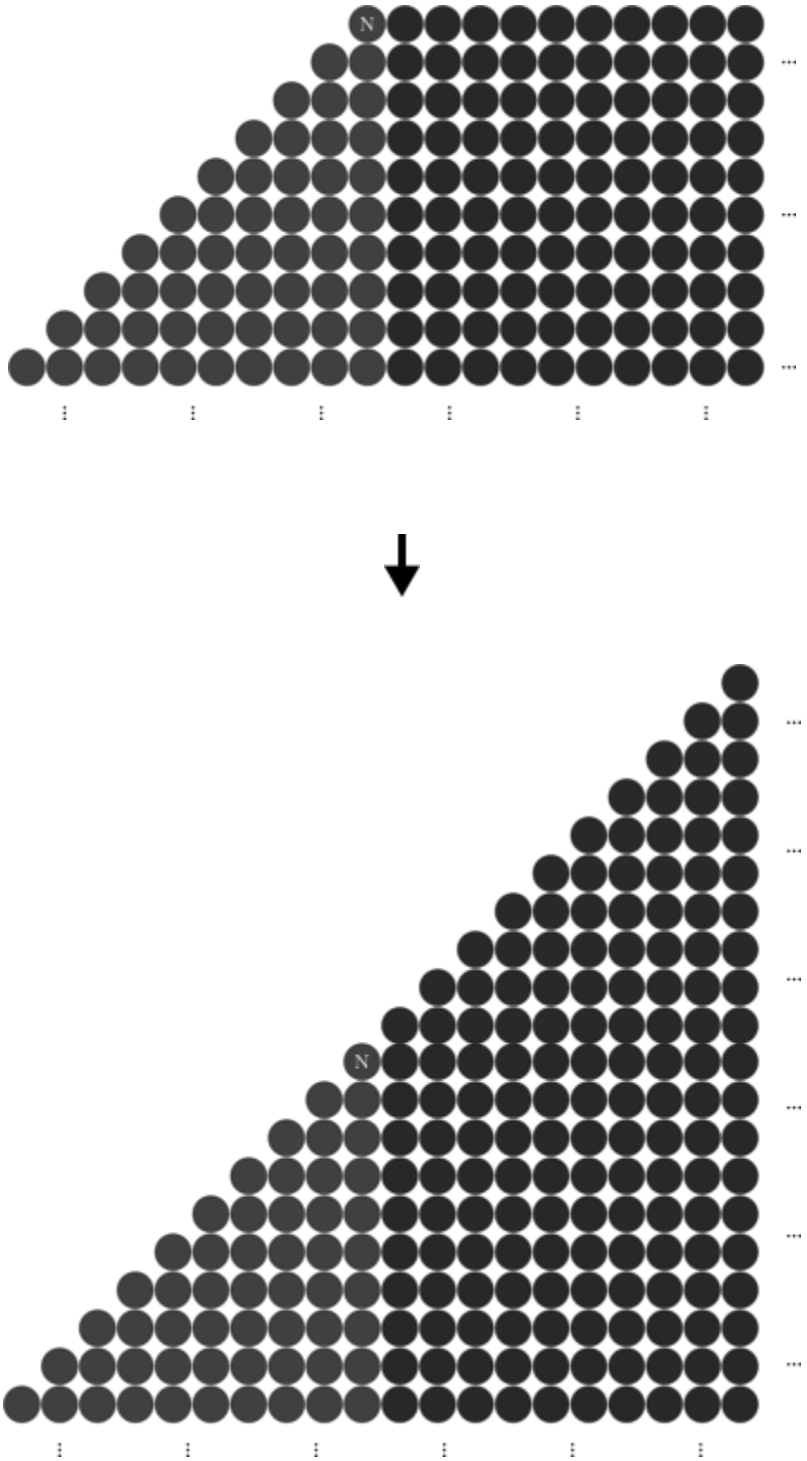


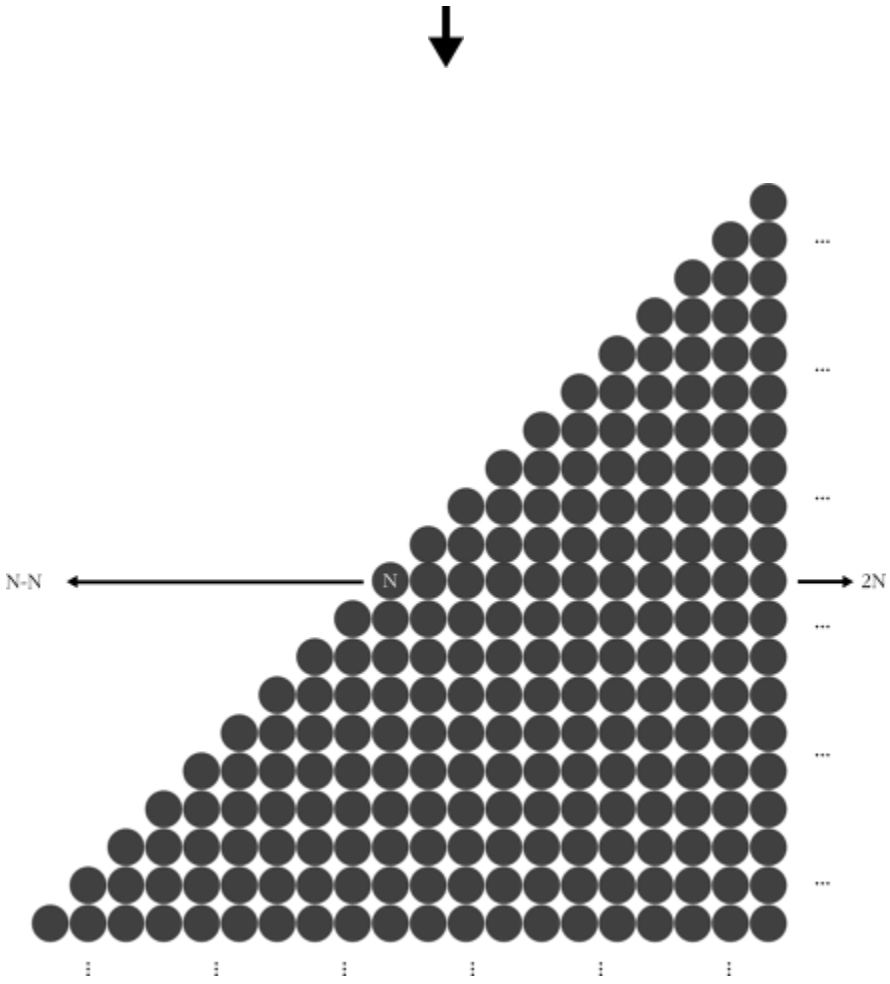

So, all we need to do is generate the number, flip the number line upside down, then invert the gaps and numbers.




Now we have a boundlessly infinite number to study. From here, all we need to do is make N number of copies to the right, then add progressively more numbers to the right of this to make numbers

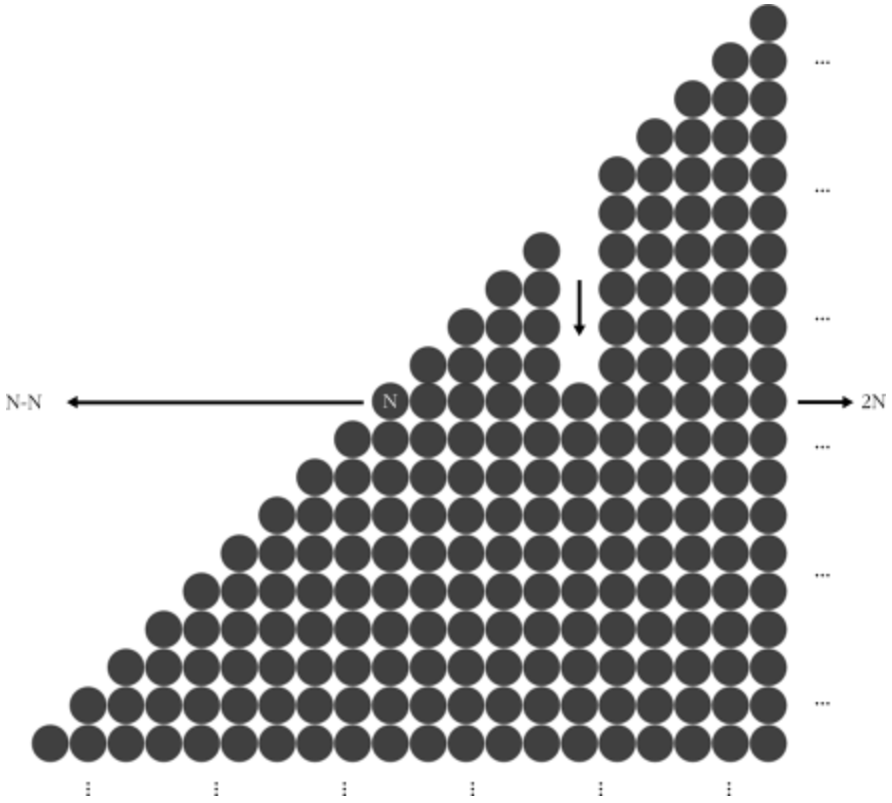
greater than N, and subtract numbers to work all the way to 0 (N-N), creating a number line that is at least 2N long.






Another implication of the fact that cardinality is the same as magnitude in this arrangement of numbers is that we can also study N by rotating our original line 90 degrees clockwise and flipping it along a horizontal axis to make the numbers ascend from left to right. I thought this second method was somewhat confusing, so I decided against creating a separate set of graphics for it. It results in the same image above. (DRAFT NOTE: Let me know if future drafts should include it, however.)

Of course, one can still object and say that numbers past N just equal N, just like $\aleph_0$. After all, we can just push every number past N downwards and it looks like each digit will biject with something new just like the old system.




This doesn't work, however, as numbers in this line decrease from N to N-N (i.e. 0). Meaning that the digits of this number ((N+5)-5) (remember, the dots are just numbers in unary) must have a lower bound at zero. Pushing N+5 down 5 units, for example, will just push N+5's lower bound to five spaces below that 0 point, or -5, where no number in this particular number line has digits below that point.

(DRAFT NOTES: Let me know if I would need a graphic for this.)

A system of counting numbers with numbers of infinite, yet specific magnitudes isn't really such a foreign concept, even without having to travel beyond the classical naturals. To find one, let's consider $\boldsymbol{\pi}$ as it appears on the hyperreal line.

We can count from r1 and follow it all the way to $\pi$. That number would be of an immensely suprafinite number of 1's all aggregated to make what, when represented in base 10, is 3,141,592,653,5..., with suprafinite numbers with precise values to the left and right of it.

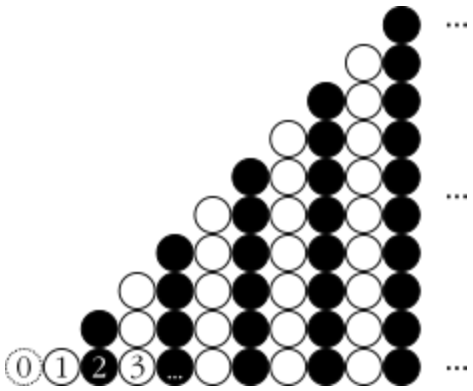
I realize that this contradicts the proof that there are as many points in an interval on the real number line as there are in the entire real number line. Such a proof would not work using strict counting methods.

(DRAFT NOTES: Should I put a demonstration as to how this proof would fail?)

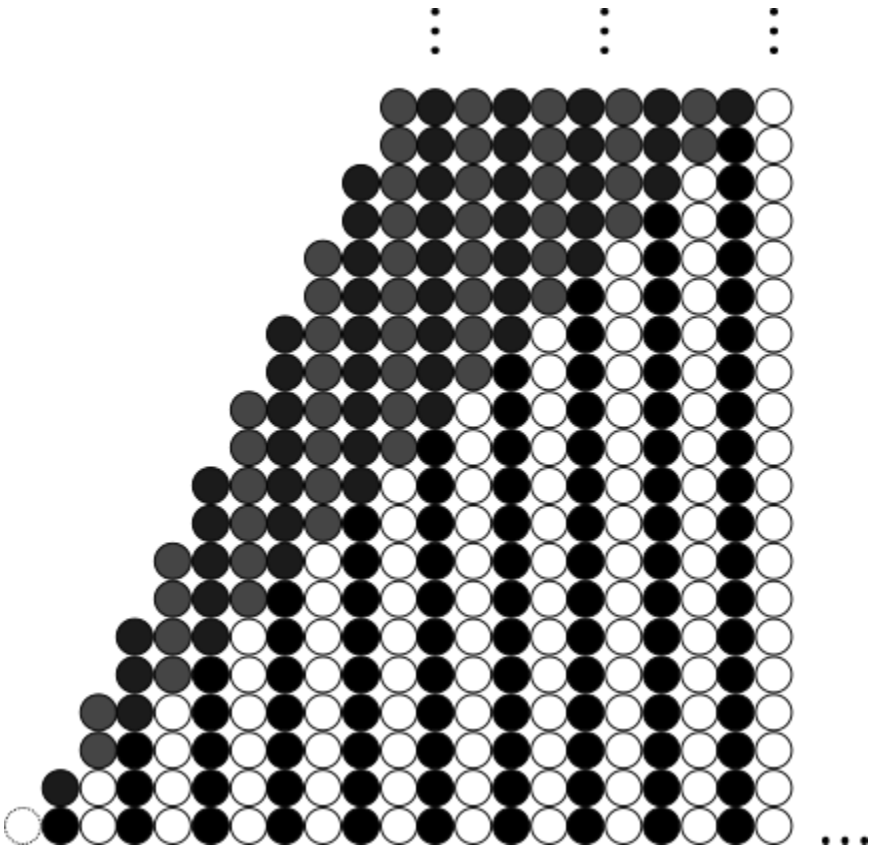
To really prove all this, however, we'd need to show the following:

1. Every number, no matter how infinite, will always have a lower bound at zero and an upper bound at its leading digit.
2. Due to the lack of a limit on the size of numbers, every attempt to create an infinite set of ascending numbers will always have a largest number that has a unary digit for every element in the set. From now on, this number will be referred to as the superlative.
3. There is no clear line between finite, semi-infinite, and infinite sets, just as there is no clear line between finite, suprafinite, superlative and beyond.
4. It is possible to journey from the lowest value in an infinite set to the superlative and beyond in a strictly trackable way.
5. Every infinite set is nested within larger infinities that vary by finite or suprafinite amounts.

We can demonstrate this with the experiment below. To begin, let's return to our original number line:




Assuming we won't be able to find an upper bound to whatever this leads to, and the totality of the numbers in this set equal a classically infinite cardinal N that won't appear on the line. Since we can't add any more numbers to this line, all we can do is make a copy of it. Rather than separate this line on a three dimensional space, we will stack them on top of one another:

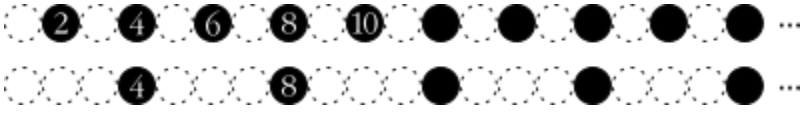


So now we have a line where its magnitude is twice its cardinality. Keep the bifurcation point between the two stacks of numbers in mind as we will compress it to look like this:

This is to show what happens when every number in a set of N size is doubled. It will also show a bifurcation point between numbers of infinite size, meaning the second number that's stacked on top of the first will have a lower bound at the upper bound of the bottom number, and the bottom number will have a lower bound of zero. As such, we would know that the upper number also has an upper bound, no matter how infinite that number gets.

The next line we will make is a line that should reach twice the distance of our original line. To model this, we will repeat every number like so:

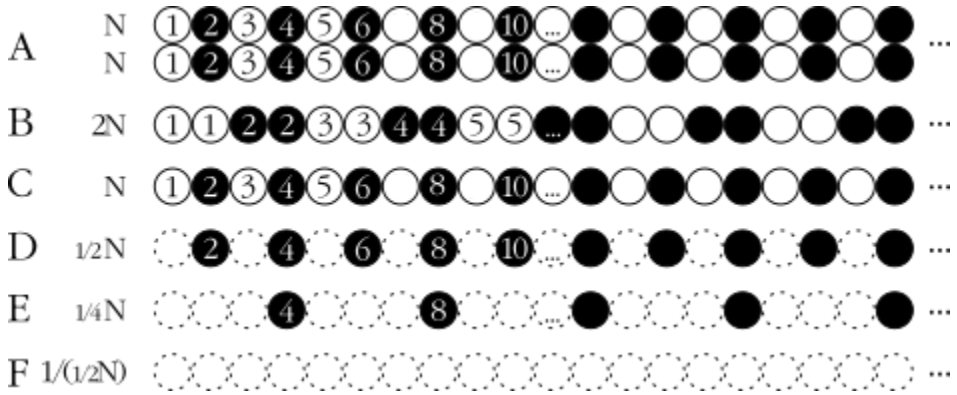
The next line we will make is just our basic number line. We know what that looks like, so let's skip to the next two lines, which have the evens and multiples of 4:



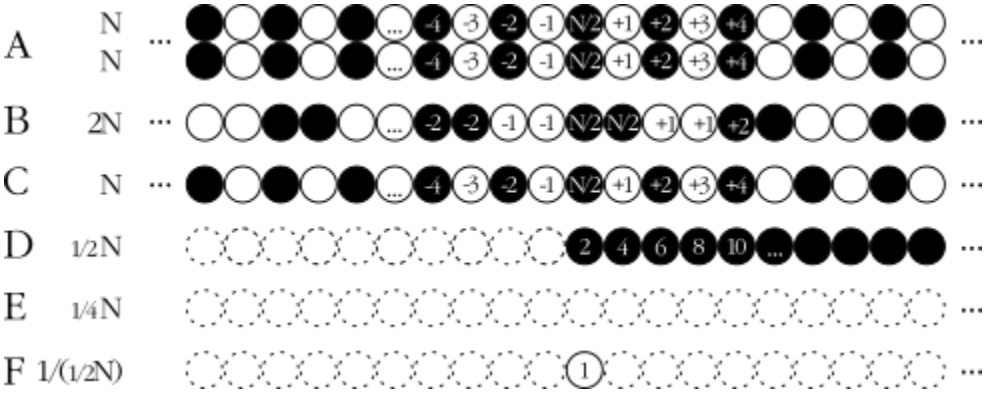

These two lines will show the behavior of semi-infinite sets. They will also be what we will be tracking in our journey from the beginning to the "end" of this particular number line.

The final line (Line F) is a set that is a fraction of the total line, but has a suprafinite number in the denominator (1/ ½N). This will produce a set with only two elements:

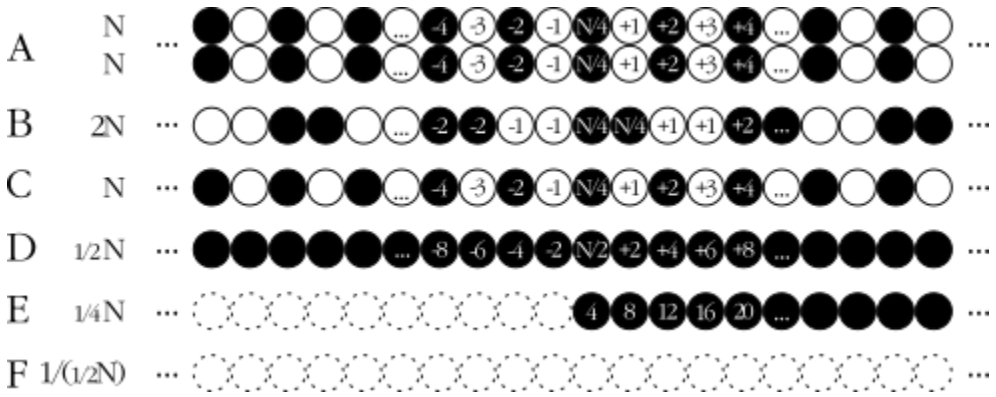
This will also help to demonstrate that there is a smooth transition between infinite and finite sets and numbers. And here are all our number lines together:



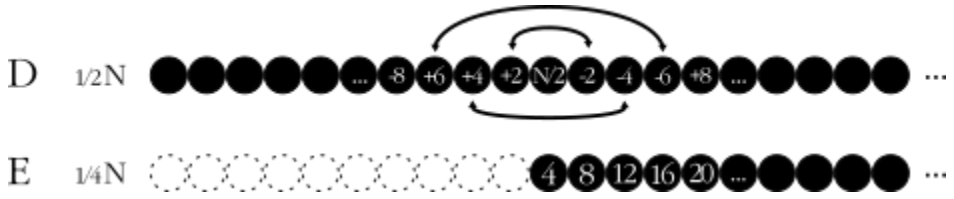

From here, we are going to push the evens and the multiples of 4 to the right. As noted in the first thought experiment, the evens will only make it halfway to N. We will examine this point first. As predicted, our 2 is paired up with a number that is half the size of N. The magnitude of this number is also equal to the cardinality of this particular set of even numbers.



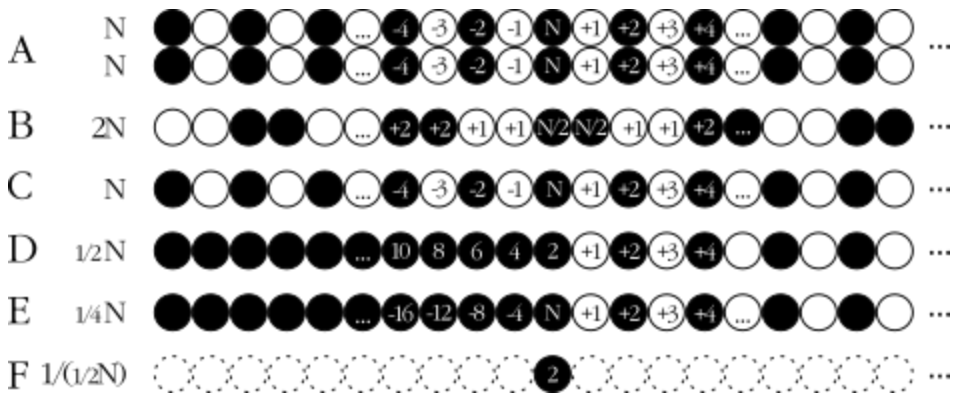

From here, we can travel over to where Line E starts:




In addition to Line E being there to show how semi-infinite sets smoothly transition into finite ones, it will also act as an axis. Numbers on the ½ N line  to the left of it will be swapped to numbers on the right of it. The point of this is that this is the quickest way to bring the number 2 in our even number line over to meet N while keeping with our requirement to strictly track the positions and magnitudes of every number. The other way would require a list of sets with an infinite number of entries.




Now that we've used a method to traverse an infinite set that maintains a strict "chain of custody" between 2 and the superlative that was posited to exist, we now have justification to believe the behaviors that we see at this point.




Line A shows a number with a magnitude of 2N, outside the range of N. If we were to slide each individual number in line A to match their position on Line C, the 2N of line A would line up with 2N in line C, outside the range of N, once again lining up with the expected behavior of this system.

Line B shows that it has only progressed to ½ of its full length, eventually terminating at 2N.

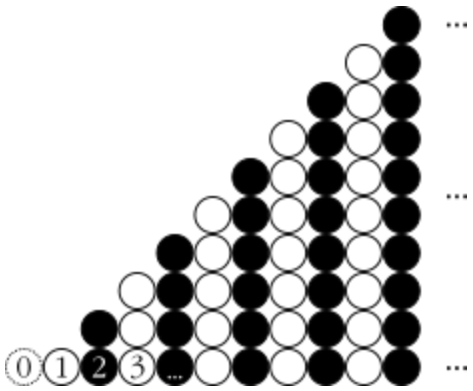
Line C shows that we've traversed to the superlative of our line and the numbers beyond it.

Line D and E both show their expected values as well.

Finally, at Line F, we get the second element of our two element set.

The larger point is that when we try to make an infinite cardinal like $\aleph_0$ using conventional methods, all we end up doing is making a line up to some suprafinite point that is nested in a larger gradient of infinities beyond it. The upside is that we can do precise mathematics with numbers, no matter how infinite they may be.

But none of this is what we're talking about when we talk about $\aleph_0$. We want to create a cardinal that is equal to the entirety of the suprafinite numbers. To once again try to do so, let's return to our basic number line:




The dotted circle labeled “0” isn’t zero. It’s a placeholder for the absence of numbers. It’s helpful if you want to create something like the real number line, where numbers are designated by their distance from zero. But that’s just a position on the line. It isn’t the absence of the line itself.

In other words, If we were to try to count zero by putting a “1” there, all we did was count one placeholder, not the absence of numbers. We’re concerned with the behaviors of numbers, not ordinals, so this isn’t helpful.

In fact, the only number this system can’t model is zero. If this sequence of numbers were theoretically complete, taking into account all possible mathematical calculations we could ever perform on the line, this $\aleph_0$ equivalent can only really be described as the opposite of zero. Rather than trying to define a particular value on the line, it is more useful to define this theoretical infinite cardinal as what it isn’t: negative zero. I’ve chosen to write it as !0 because -0 is just going to cause confusion when it appears in formulas. (DRAFT NOTES: If anyone has any thoughts on how to represent it, please let me know.)

This is about as close as I’ve gotten to the typical construction of an infinite set. However, this number is just the biggest something can get when it is relative to one particular zero, not all possible zeros. In other words, you can still make copies of it and math works the same regardless.

$$!0 + !0 = 2!0$$

!0 can also have numbers that are greater or less than it by classically finite amounts:

$$!0 + 1 = !0 + (!0 - (!0-1)$$

One can still perform math with these numbers in a precise way, but we're experiencing the same problem as the last attempt at fencing in these numbers. The existence of a number with a magnitude of 2!0 also implies the existence of every number between it and zero. Why are infinite sets behaving like finite ones? Essentially it boils down to this:

1. My experiments have shown that when attempting to create a set of numbers that grow sequentially, the magnitude and cardinality always match, meaning the largest a number can get in that set is equal to the superlative, and there is always a direct line of ascension between zero, the superlative and beyond. It does not matter how infinite in size the set may be.

2. Because numbers can be of any size, some kind of suprafinite math can always be done with the numbers within the set to meet or surpass the superlative of any set.

3. When we travel to the limits of any infinity using suprafinite math, we will find that the infinity we were constructing is always nested inside a gradient of ever increasing infinite sets that vary from one another by finite or suprafinite amounts.

4. In other words, it is impossible to construct a set of numbers such that those, and those alone, are the supranaturals.

5. Therefore, the totality of supranatural numbers is more accurately described as a class.

Let's now review each of the paradoxes, scenarios and experiments in this paper to see how the elements of this system were hinted at.

1. The magic ant paradox
    a. Hinted at the existence of semi-infinite sets.
    b. Hinted at the existence of superlative numbers.

2. The interval argument counterexample
    a. The bifurcation point between the evens and the gaps suggested the existence of semi-infinite sets and superlative numbers.
    b. The right-side evens line suggested a way to count by twos to a number that is twice the size of the naturals. This hinted that math could be done with infinite-sized numbers and sets in a precise way. It also hinted that numbers can't be contained in any one set.

3. The diagonal argument counterexample
    a. Showed that simple bijection was an ineffective method for gauging the sizes of infinite sets.
    b. Hinted at the lack of a concrete boundary between the finite and infinite

4. The integer method for counting the reals
    a. Showed that this was an ineffective method to gauge the size of the integers
    b. Hinted that $N < I$

5. Scenario 1:
    a. Showed that, theoretically, one could precisely track the sizes of an infinite number of subsets of P(N). This is just conjecture, however.

6. Scenario 2:
    a. Showed that one could count anything with Cantor's methods, even if it were illogical to do so.
    b. Hinted that $N < Q$

7. Scenario 3:
    a. Hinted at the existence of superlative numbers.
    b. Demonstrated that there isn't a clear line between different infinities or between the finite and infinite.

8. N embedded in a line:
    a. Sorting the numbers on the line in order of magnitude hinted that it's not possible to contain something truly boundless in a set.

9. Triangle paradox:
    a. Showed that current notions of cardinality can be used to break geometry.

10. Cantor's proof of the countability of the rationals
    a. This cast doubt on whether $N = Q$.
    b. Hinted that Q's cardinality should be derived from N and not have a cardinality on its own.

11. Revisiting Euler's argument

a. Reinforced the conclusions of the magic ant scenario.

12. My attempt to get closure to work
    a. Suggested that finite numbers could be multiplied by suprafinite numbers to surpass the superlative of a set.
    b. Suggested that suprafinite numbers could be multiplied by finite numbers to surpass the superlative of a set.

(DRAFT NOTES: Please note that from here on, I'll be referring to the supranaturals as "naturals". I only used the name previously to differentiate it from the classical naturals. If this system becomes the norm, that is how I expect them to be referred to, and continuing to refer to them as "supra" doesn't seem to fit. Let me know if the norm would be to keep the naming convention throughout.)

Replacing the closure property with a co-emergence property resolves every anomalous behavior we've encountered, but the co-emergence property does create some anomalous behavior of its own. At the set level, the questions about whether there are as many even numbers as natural numbers and whether every number can be doubled are split into two separate problems.

- When we ask, "Are there equally many even numbers as naturals?", we should be asking, "What happens when we halve the cardinality of a particular set of ascending numbers?"
- When we ask, "Can every number be doubled?", we should be asking, "What happens when we double the magnitude of every element of a particular set of ascending numbers?"

I do admit this is something of a cop-out. The question is still just pushed to the class level. The conclusion that I eventually came to is that it's a non-question unless there's something to measure, and if there's something to measure it defaults to the behaviors above.

Nevertheless, I still tried to answer the question, but it's outside of the scope of this paper. My full thoughts on the matter of the relative sizes of classes (as well as my construction of real numbers) lean into the phenomenological aspects of numbers, and while my answer satisfies me, it may seem a bit too close to numerology for something that's supposed to have proofs based on it. I'm sure there are other set theories that can handle this better than myself. This section of the paper was just a proof of concept to show that a system of counting numbers could be made that was consistent with my findings.

If it turns out there is an appetite for "phenomenological mathematics" (I'm not quite sure what to call it), I would be happy to write more on the subject, which will probably be after I get some kind of job that won't kill me before my mom dies. I can't waste any more time on this pipe dream.

## CONCLUSIONS

This paper has shown that Cantor's methods can be used to count anything, even when it contradicts Cantor's theorem. The concept of order types was shown to not hold when observing a countably infinite number of objects at once. The closure property of the naturals was then falsified and a system of numbers was constructed that did not depend on it. This model is compatible with Cantor's theorem, even though it is redundant. The co-emergence property, along with the smooth transition between finite and infinite allows math to work precisely at any scale. Lastly, it allows for a single set of rules to govern finite and infinite sets.

Assuming my system gains wide adoption, there are some open questions that will need to be addressed, though I'm not sure if some of them can be answered:

1. Are there primes of suprafinite magnitude, or do primes get so dense that all numbers become divisible by something after a certain point?
2. If there are primes of suprafinite size, then we can position them starting at r2 on the hyperreal line. All known primes would fall inside a tiny sliver of the number line past 0. Is there a way to project where we would expect primes to be beyond this point in a holistic way that covers the entire number line?
3. Does R still equal P(N)?
4. If numbers can smoothly and precisely reach infinite magnitudes, do irrational numbers exist?
5. Is there a way to create a "prototypical" infinite set from which all infinite sets derive? I'm guessing no, since this system recognizes no boundary between the finite and infinite.
6. Is there a way to make a true hierarchy of infinities with clear borders between them, as they exist in ZFC?

I also expect that questions of cardinality can be handled with fractal math. I can also think of a couple of exotic sets a person could make, though those thoughts aren't particularly well formed.

This brings us to the matter of what happens to current work that depends on Cantor's work and the closure property. As a layperson, I can't really give much in the way of a definitive statement on the subject, but the damage seems to be nothing short of apocalyptic. Beyond set theory, everything from Peano arithmetic and ring theory to real analysis depends on them.

A dry recounting of the consequences of this paper doesn't seem adequate, and not just because it doesn't really give a sense of scale as to what will be affected. There are human beings behind every proof– brilliant minds in pursuit of mathematical truth at the edges of the ineffable, and so much of their work will need to be reworked or discarded. I can't imagine the heartbreak at seeing Cantor's beautiful hierarchy of infinities replaced with something so painfully intuitive and boring.

Worse still, anti-intellectual types won't hesitate to ridicule mathematicians for "not being right the first time", as if mathematics weren't an ongoing process of discovery. The fact of the matter is that Cantor and his intellectual heirs just took a system of premises and followed them to their logical end. Their work is no less magnificent now than it was before this paper was published.

But hucksters don't care about that. They don't care that if not for Cantor, we wouldn't have Paul Cohen's forcing technique or or Godel's incompleteness theorem. They don't care that Cantor's mark can be found nearly everywhere in modern mathematics, and by extension, the technological and intellectual advancements that mathematics has brought us over the past hundred years. They won't care that I left a few things out of this paper because I didn't consider them rigorous enough to publish, because their aim is to distract their marks from their own lack of rigor.

This shouldn't be a cause for despair, however. People will do what they want, and the best we can do is be prepared. For those of us who find worth in genuine, rigorous intellectual pursuit, this paper is just a single step in a larger journey. Though there may be less to explore in set theory, a less formalistic approach to mathematics may allow us to visualize numbers in a new way, bringing about avenues of exploration that we may not have thought of otherwise.